Mathematics Extension Groups at Melbourne High School

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Melbourne High School (boys only) is one of two select-intake government schools in Victoria where students achieve outstanding results across all studies. As the coordinator of the High Achieving Student Program I have had the privilege of working with the mathematically elite in the school at Years 9 and 10 for many years. I have devised materials that are undertaken by students chosen to participate in the program with such questions as “if a capacity crowd at the MCG was to be evacuated onto the playing surface of the ground – would they fit” given no dimensions nor formulae is a favourite and frees up the minds of students to discussion/argument. All aspects of the program will be discussed.

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Melbourne High School (boys only, years 9 to 12) is one of two select-intake government schools in Victoria. Melbourne High School's curriculum is derived from the fact that at year 9 students arrive after satisfying the standards of the entrance examination at a number of different albeit a high set of different levels. The general curriculum offering at years 9 & 10 is designed to address transition issues and is based on the expectation that students will have the ability and the desire to be challenged through a broad education. It is school policy that students not be accelerated beyond their current year level, that is year 9 & 10 students study concepts suitable to those levels, however given the level of performance of the students at these levels the mathematics faculty supplies additional enrichment material throughout the courses designed to apply the concepts to more complex tasks. This is delivered via the text series: Maths Dimensions 9 and 10 (Pearson) where an enrichment section follows each chapter.

In addition the mathematics program targets the top 10% of the students in years 9 & 10 to undertake additional tasks of a more challenging nature. These "high-achieving" students are presented with a program of non-routine tasks and the groups are called the year 9 and year 10 maths extension groups respectively.

The Mathematics Extension Groups

As a lover of mathematics and the language of logic the opportunity to work with students of a high calibre is a fantastic opportunity. To work with these students where they argue and fight over the finer points of mathematical thought is a special experience.

Picture a room full of students from years 9 or 10 – assembled separately once a fortnight, working – thinking – arguing – seemingly fighting (intellectually) and there you have it. The students engage in a mathematical jousting match either teaming up with others or working alone to tackle the current fortnightly challenging problem(s). For these boys mathematical conundrums are the vehicle for discussion, exploration and learning. A difference of mathematical opinion is an occasion for open debate – a free for all – a time to convince other participants of the validity of the logical conclusions to their solution. This is the ultimate meeting of mathematical minds where all are challenged to reach their potential.

Structuring the Mathematics Extension Programs

Operating the year 9&10 programs requires the selection of appropriate students, to find a time that suits all participants, to provide interesting material and finally to have the energy and belief to oversee such a program.

Selecting the students

Student selection into the program needs to be fair and open to all students in the year level. It must not be subject to any form of favouritism. A method of selecting students to participate in the program is done by

- undertaking a series of tests – IQ, reference to entrance exam data, other externally derived tests
- teacher recommendation – this is a very accurate indication but needs to happen after a number of weeks in to the semester. The mathematically elite students show themselves to their teachers in the classroom environment - it is the most efficient way to find them.

Finding a time for the program

In general school programs are under continual pressure, where the time required for additional programs such as drug education, pastoral care, work education, work experience, sex education, driver education and so on impinge on the time that mathematics occupies in the curriculum. Time for a program such as this can not easily be found within the normal teaching program. At Melbourne High School the only time to run this program is a negotiated lunchtime which suits all selected students.

Having students commit to the program

Students can start off well committing to the program, however this initial enthusiasm can ebb away. Students need to commit to the full program and this is ensured by the completion of a contract whereby the supervising teacher and each student and their parents sign an agreement detailing aspects of content, assessment requirements and the date or timing of the program.

SAMPLE CONTRACT

Melbourne High School High Achieving Students Program, 2007

STUDENT NAME: Thomas Ng-Santomartino FORM: 10H

SUBJECT/COURSE AREA: Mathematics

SUPERVISING STAFF MEMBER: Mr Bull

The content to be covered

Thomas has been selected into the Year 10 Maths Extension Group which will meet once a fortnight at lunchtime during Terms 2 and 3. Eight challenging questions will be distributed during these sessions covering a wide variety of unusual and non-routine Maths topics.

Assessment method

To obtain a satisfactory grading:

- A complete response to the eight extension questions needs to be submitted including a reflection statement for each task.
- All work is to be submitted in a bound workbook with an index showing the placement of all questions undertaken.
- Parents/guardians need to sign the beginning and end of each task in the workbook to show that they have seen the work undertaken by the student.
- The supervising staff member the student and their parent/guardian need to provide comments on the completion and reflection of their work both before and at the completion of their work and sign the following relevant sections.

Date for submission of work

All work needs to be submitted at the end of Term 3.

BEFORE WORK ON THE UNIT OF WORK

STUDENT STATEMENT:

I am pleased to be selected into this group of the best maths students in Year 10 and will work through all the work that is expected.

Signature:

PARENT STATEMENT:

We will track the progress of Thomas as he works through the tasks given to him.

Signature:

AT THE CONCLUSION OF THE EXTENSION UNIT:

STAFF COMMENT:

Thomas Ng-Santomartino *completed all the questions required on time showing a full range of sophisticated methods of solution. Well done, you are a talented student and your enthusiastic approach shows that you enjoyed the challenge.*

STUDENT COMMENT:

The work was at times challenging but I enjoyed working through each question.

PARENT COMMENT:

Thomas Ng-Santomartino *worked through this work with us looking over it. We are pleased to see the school providing him with work of a more difficult nature and have looked through his bookwork.*

Material

The material presented to the students parallels the concepts and the levels covered by the students in their mathematical studies. With the program operating during terms two and three the first section deals with topics covered during the first semester, whilst the second section deals with the topics studied in the second semester. The tasks have been and are being developed over a period of time and have been sourced from a variety of materials – maths competitions, enrichment materials, problem solving books, interesting questions taken from maths texts and so on. A list is included at the end of this paper.

Formulating Tasks

Mathematical tasks are around us all the time. The more traditional ones are sourced from texts and competitions and altered to suit the level of the students. The more interesting tasks are the ones that we stumble on. An example of this type of task was conceived as I sat at the MCG listening to the emergency evacuation announcement.

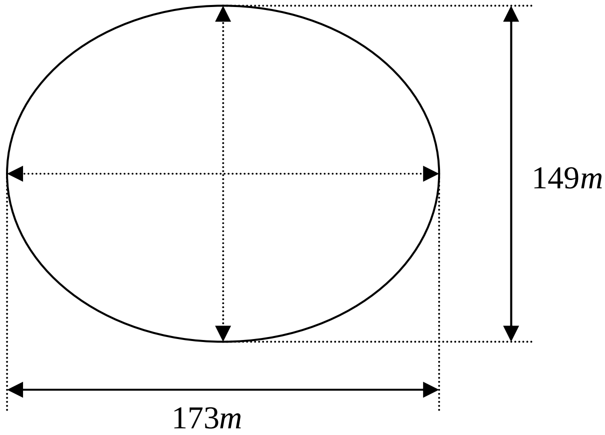
This is a favourite task which is presented to the students at the start of the program. It would be unlikely that all the stands at the MCG would be affected in this way but the task is unusual and accessible which is based on measurement concepts. No information is given – the students need to estimate all measures – the capacity of the ground, the dimensions of the ground and the formula for the area of an ellipse. This is where team work in groups pays dividends.

The students glue this task into their workbook and are then given twenty minutes to make preliminary explorations of the task either in groups or on their own. The workbook is then ruled off and the students are able to complete work on the project in their own time before it is finally sighted and signed off at the next fortnightly session.

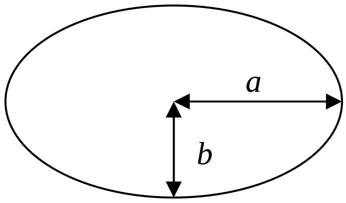
Task 1
If there was a disaster at the Melbourne Cricket Ground on Grand Final Day and the people were asked to leave all their possessions and stand on the ground, would they fit?
Draw a diagram and estimate the dimensions involved and justify your answer.

Solution: Task 1

The initial estimate required is that of the dimensions of the ground. The fifty metre line marked at each end of the ground could be used as a guide.
The Melbourne Cricket Ground is in an elliptical shape as follows:



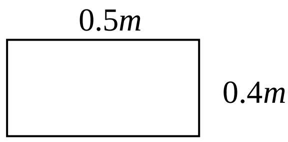
The area of an ellipse is as follows:



Area = πab

The accurate area of this ellipse is:

Area = $\pi \times 74.5 \times 86.5 \approx 20245 m^2$



The next question is then to estimate the number of people that could fit into one square metre. Assuming that the average person is a rectangle:
then it would perhaps be reasonable to suggest that 4 people could fit into one square metre. This would certainly be true if some of the people were children, although the issue of children keeping still whilst packed tight on the ground with the emergency going on around them could be debated.

Theoretically then, $4 \times 20245 = 80980$ people.

Mathematical arguments as to the reasonableness of this estimate as to how the people could be assembled on the surface to enable such packing are invited and would need to be made in order to justify the solution.

Case Study – formulation of an Extension Task, Task 2

Often in our teaching of classes at a higher level interesting questions are raised which can be adapted to tasks of lower levels.

This task grew out of a question asked by a Year 11 student whilst working through the topic of sequences and series in a General Specialist Mathematics class from the text: General maths Dimensions – an advanced course, Bull & Nolan (Pearson Education). Often questions and tasks examined at a higher level can be repackaged into a task suitable for use in the Maths Extension Program.

1. An arithmetic sequence is a set of numbers which continues in a pattern by adding a constant difference to generate the next term. The terms of an arithmetic sequence with a first term of a and a common difference of *d* is: {*a, a+d, a+2d, a+3d,* …..). The symbols for the first term is t_1, t_2, t_3 with t_n being the n^{th} term in the sequence.

(a) Show that the n^{th} term of an arithmetic sequence is given by $t_n = a + (n-1)d$, hence find an expression for (i) t_{n+1} (ii) $t_{\frac{a}{d}}$ (iii) $t_{-\frac{a}{d}}$.

The sum of *n* terms of an arithmetic sequence is given the symbol S_n. The general rule for the sum of *n* terms is: $S_n = \frac{n}{2}(2a + (n-1)d)$.

(b) The sum of an arithmetic sequence is $2n^2$. Using the methods (i) of generating the sum of terms in the sequence and (ii) using algebra find the values of *a* and *d* hence find the first three terms of the sequence.

Solution:

(a) $t_1 = a,\ t_2 = a + d,\ t_3 = a + 2d,\ t_4 = a + 3d, ...$ so by observation the coefficient of *d* is one less than the term number, ie *n*. Hence $t_n = a + (n-1)d$

$(i)\ \ t_{n+1} = a + (n+1-1)d\ \ \therefore t_{n+1} = a + nd$

(ii) $t_{\frac{a}{d}} = a + (\frac{a}{d} - 1)d,\ \ \therefore t_{\frac{a}{d}} = a + a - d,\ \ \therefore t_{\frac{a}{d}} = 2a + d$

(iii) $t_{-\frac{a}{d}} = a + (-\frac{a}{d} - 1)d,\ \ \therefore t_{-\frac{a}{d}} = a - a - d,\ \ \therefore t_{-\frac{a}{d}} = -d$

(b) This produces an interesting mathematical scenario.

(i) $S_1 = t_1 = 2(1)^2 = 2$

$S_2 = t_1 + t_2 = 2(2)^2 = 8,\ \ \therefore t_2 = 8 - t_1 = 8 - 2 = 6$, so the sequence is (2, 6)

$S_3 = t_1 + t_2 + t_3 = 2(3)^2 = 18,\ \ \therefore t_3 = 18 - (t_1 + t_2) = 18 - 8 = 10$, so the sequence is (2, 6, 10)

Hence $a = 2$ and $d = 4$

(ii) $S_n = \frac{n}{2}(2a+(n-1)d) = an + \frac{dn^2}{2} - \frac{dn}{2} = 2n^2$

Equating coefficients:

$n^2: \ \frac{d}{2} = 2, \ \therefore d = 4$

$n^1: \ a - \frac{d}{2} = 0, \text{ for } d = 4, \ a - 2 = 0 \ \therefore a = 2$

The sequence using a first term of a with a common difference d of 4: $(2, 6, 10)$

In this task for part (b) the Year 11 student approached the question from a formula approach – because for him the solution of Maths problems was based about the application of a suitable formula.

Reaching the step: $an + \frac{dn^2}{2} - \frac{dn}{2} = 2n^2$, with one equation and three unknowns this working step stopped this student in his tracks. I showed him the Part (b) (i) approach which required him to think of the information in a different way – using a problem solving approach and then wondered why his algebraic approach had met a dead end. I then saw the algebraic line in terms of powers of n and achieved a solution by equating the coefficients of n as shown in the solution above.

This example was presented to the class where they were lead through thinking processes highlighting where if a problem can be appreciated from a various views then multiple solution processes can be applied. This led to a discussion of the power of mathematics as a problem solving tool – an algebraic or apply the formula approach is a disappointing simple method which most students depend on to solve the type of questions that we often manufacture.

Often in the traditional, mainstream classroom with time restrictions and so on Mathematics is reduced to the use of formulae. The strength of this Extension Program is to present students with a series of non-routine, thought providing experiences which are intriguing and require the students to be adaptable thinkers according to the problem situation.

Resources

These are best collected over time from a variety of sources, both local and overseas. A short list might be:

Maths competitions such as

- Australian Maths Competition – Westpac
- University of NSW: ICAS
- Maths Challenge: AAMT
- Maths enrichment stages: AAMT
- IBM – Melbourne University Maths competition

Useful reference materials

- Set of books: *Can you solve these* – Tarquin publications
- Edwards, King O'Halloran (….). *All the best from the Australian Maths Competition.* Australian Mathematics Trust.
- Soifer (….). *Mathematics as Problem Solving*.
- Canadian Mathematics Competition. *Problems, Problems, Problems*. University of Waterloo.
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