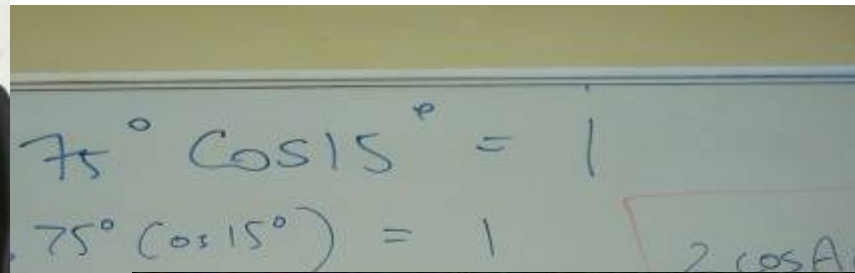
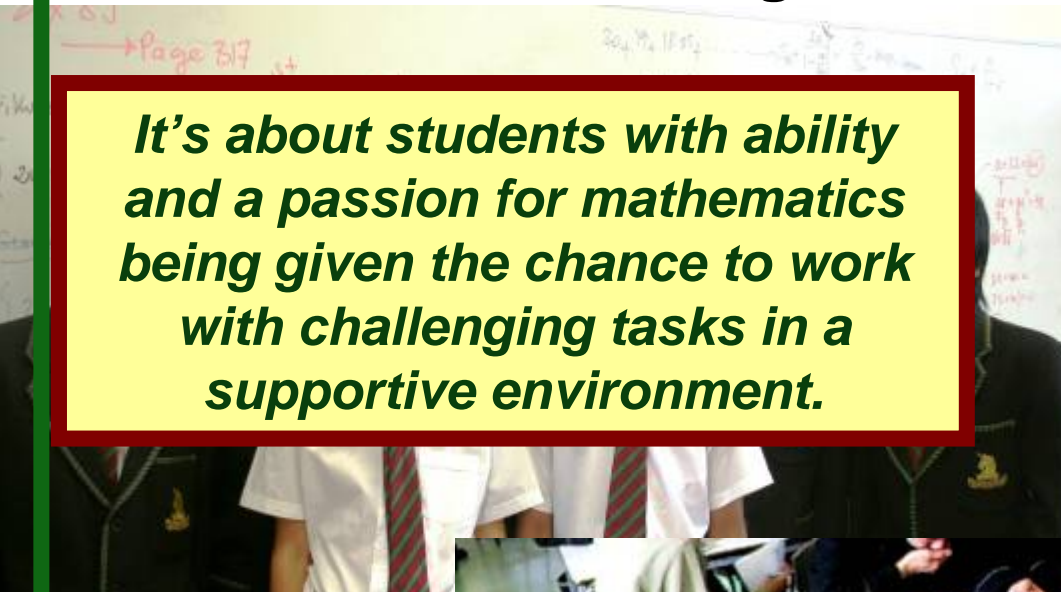


The Year 9 & 10 Mathematics Extension Groups

Ian Bull - Melbourne High School 1999 - 2008



It's about students with ability and a passion for mathematics being given the chance to work with challenging tasks in a supportive environment.



25. OCT. 1999



12:49
18. OCT. 1999

Why run a Maths Extension Enrichment Program?

- to provide a focus for mathematically capable students and acknowledge their high level of performance.
- to allow elite mathematical thinkers to have fun with non-routine questions in a supported environment.
- in a school that doesn't accelerate any students beyond their year level this program is pivotal in providing challenge.
- to provide an environment for like-minded students to gather and engage in the solution of a variety of problems.

Key issues in implementing this program

1. Selecting the students.

- how many students can take part?
- how can you find/select them?

2. Delivering the Program - finding time.

- lunchtime, after school?
- as part of an elective system?

3. Commitment to the Program - how to get the students to keep coming.

- contracts signed by the student/parent/teacher
- incentives – prizes

4. Materials, tasks to be covered.

- locating questions, tasks that can be completed in a set time

5. Marking and checking work

- finding time

Selection of Students: Teacher Recommendation Sheet



- 10 _____
- **Year 10 Maths Extension Group 2007**
- The Maths Extension group will operate during terms 2 and 3 this year – eight sessions, 4 each term. It will operate during one lunchtime per week by negotiation each fortnight.
- I need each teacher to recommend 2 to 3 students per class to attend the introductory session at lunchtime in room T7. I will provide a list of materials required for the sessions to these students prior to the first session.
- Please list the names of recommended students below:
 - 1. _____
 - 2. _____
 - 3. _____
- **Please return to me – Ian Bull, ASAP.**

- **** Year 9 Academic Award winners – 2006 (now in Year 10)**
- Use the following list as a start to locate potential participants in the program. These students are all-rounders and there might be other students in your class, talented in mathematics only who are not shown below.
- Form 10A: *****
- Form 10B: *****
- Form 10C: *****
- Form 10D: *****
- Form 10E: *****
- Form 10F: *****

The Student – Parent – Teacher contract



FORM:

- **Melbourne High School**
- **High Achieving Students Program, 2007**

- STUDENT NAME: _____

- SUBJECT/COURSE AREA: Mathematics
- SUPERVISING STAFF MEMBER: Mr Bull

- ***The content to be covered***

- _____ has been selected into the Year 10 Maths Extension Group which will meet once a fortnight at lunchtime during Terms 2 and 3. Eight challenging questions will be distributed during these sessions covering a wide variety of unusual and non-routine Maths topics.

- ***Assessment method***

- To obtain a satisfactory grading:
- Students must attend all 8 sessions or negotiate an alternative session if required.
- A complete response to the eight extension questions needs to be submitted including a 20 minute initial written response, fully worked solution and a reflection statement for each task.
- All work is to be submitted in a bound workbook with an index showing the placement of all questions undertaken.
- Parents/guardians need to sign the beginning and end of each task in the workbook to show that they have seen the work undertaken by the student.
- The supervising staff member, student and their parent/guardian need to provide comments on the completion and reflection of their work at the completion of the Program and sign the relevant sections below.

- ***Date for submission of work***

- All work needs to be submitted at the end of Term 3.

- **BEFORE WORK ON THE UNIT OF WORK**

- STUDENT STATEMENT:

- _____

Signature:

- PARENT STATEMENT:

- _____

Signature:

- **AT THE CONCLUSION OF THE EXTENSION UNIT:**

- STAFF COMMENT:

- STUDENT COMMENT:

- PARENT COMMENT:

Work is marked in the lunchtime session after the next task is given – no take home marking!!!

[illegible]

.... I wanted to make a program so that students could enjoy the buzz, engage in the experience, and get excited about mathematics

– but how does that happen ???

... just ‘cos you’re there ??



**..... where can we get some tasks ???
mathematical beauty is “in the eye of the beholder”
and is everywhere**

and sometimes gems are stumbled across, even discovered
sprayed on
a bridge on the
Tullamarine
Freeway.

always keep an
eye out for an
interesting
mathematical
problem
.... like

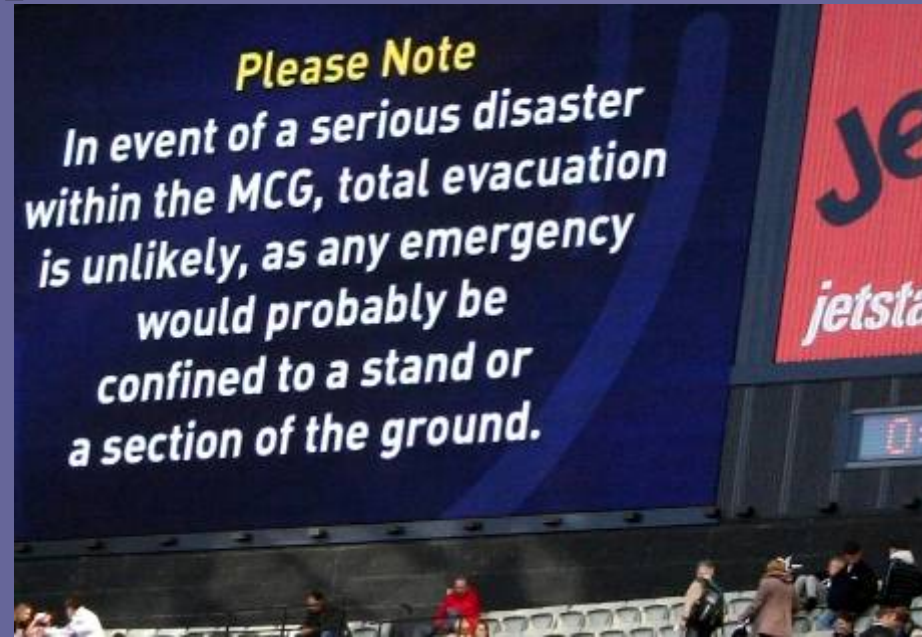


Year 10 Maths Extension Group

Challenging Question 1

If there was a disaster at the Melbourne Cricket Ground on Grand Final Day and the people were asked to leave all their possessions and stand on the ground, would they fit?

Draw a diagram and estimate the dimensions involved and justify your answer.



How did the students
approach the task ?

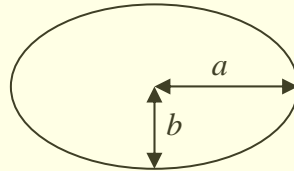
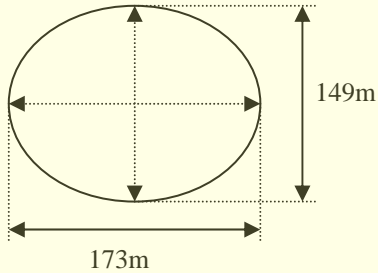


SOLUTION SECTION

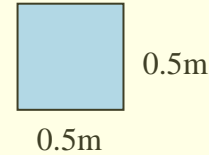
Solution Task 1

The initial estimate required is that of the dimensions of the ground. The fifty metre line marked at each end of the ground could be used as a guide.

The Melbourne Cricket Ground is in an elliptical shape as follows:



$$\begin{aligned}\text{Area} &= \pi \times a \times b \\ &= 20245 \text{ square metres}\end{aligned}$$



The next question is then to estimate the number of people that could fit into one square metre. Assuming that the average person is a square then it would perhaps be reasonable to suggest that 4 people could fit into one square metre.

This would certainly be true if some of the people were children, although the issue of children keeping still whilst packed tight on the ground with the emergency going on around them could be debated.

Theoretically then $4 \times 20245 = 80\,980$ people could fit on the ground.

Arguments as to the reasonableness of this estimate as to how the people could be assembled on the surface to enable such packing would need to be made in order to justify the solution.

④

Estimations

- ① There are 80,000 adults and adolescents
 8 adults/adolescents take up 1 m^2 - Very tight area

(I measured 1 m^2 on the floor)

Area that adults/adolescents take altogether - $10,000 \text{ m}^2$

$$\text{Calculations: } \frac{80,000}{8} = 10,000 / 10,000 \times 1 = 10,000 \text{ m}^2$$

- ② There are 27,000 children and babies with
 1 parent or more (family size)

4 children/babies/parents take up 1 m^2

(Babies are carried by parents)

$$\text{Calculations: } \frac{27,000}{4} = 6,750$$

$$6,750 \times 1 = 6,750 \text{ m}^2$$

Area that children/babies/parents take altogether - $6,750 \text{ m}^2$

- ③ There are 3,000 elderly and disabled people
 1 elderly or disabled person takes up 1 m^2

$$\text{Calculations: } 3,000 \times 1 = 3,000 \text{ m}^2$$

Area that elderly/disabled people take altogether - $3,000 \text{ m}^2$

⑤

Total Area taken on the ground:

$$\begin{array}{r} 10,000 \text{ m}^2 \\ + 6,750 \text{ m}^2 \\ + 3,000 \text{ m}^2 \\ \hline 19,750 \text{ m}^2 \end{array}$$

$$= 19,750 \text{ m}^2$$

There is still 540 m^2 space left

$$\begin{array}{r} 20,290 \text{ m}^2 \\ - 19,750 \text{ m}^2 \\ \hline 540 \text{ m}^2 \end{array}$$

Bull
 2/5/07

If there was a disaster at the Melbourne Cricket Ground on Grand Final Day and the people were asked to leave all their possessions and stand on the ground, they would fit. The total area that the 110,000 people will take is $19,750 \text{ m}^2$ and the area of the ground is $20,290 \text{ m}^2$.

Theoretically, the 110,000 people would fit on the grounds standing. Practically, there would never be 110,000 going onto the oval because there are outside exits which are most likely to be closer to the people.

Reflection

By completing this task, I was able to learn how long 1 metre is in real life as well as how much space is taken up by 1 m^2 . Also, I now know that everyone in the stands will be able to fit onto the oval.

... it's a type of chemistry that happens with the boys in this group ...

- give 'em a problem, tell 'em they're special and off they go – totally absorbed
- better still, as soon as someone needs convincing then they're all in there
- just stand back and let it happen, the discussions are illuminating



Tasks can be sourced from a variety of sources – even from a school in Hong Kong

Year 10 Maths Extension Group

Challenging Question 5

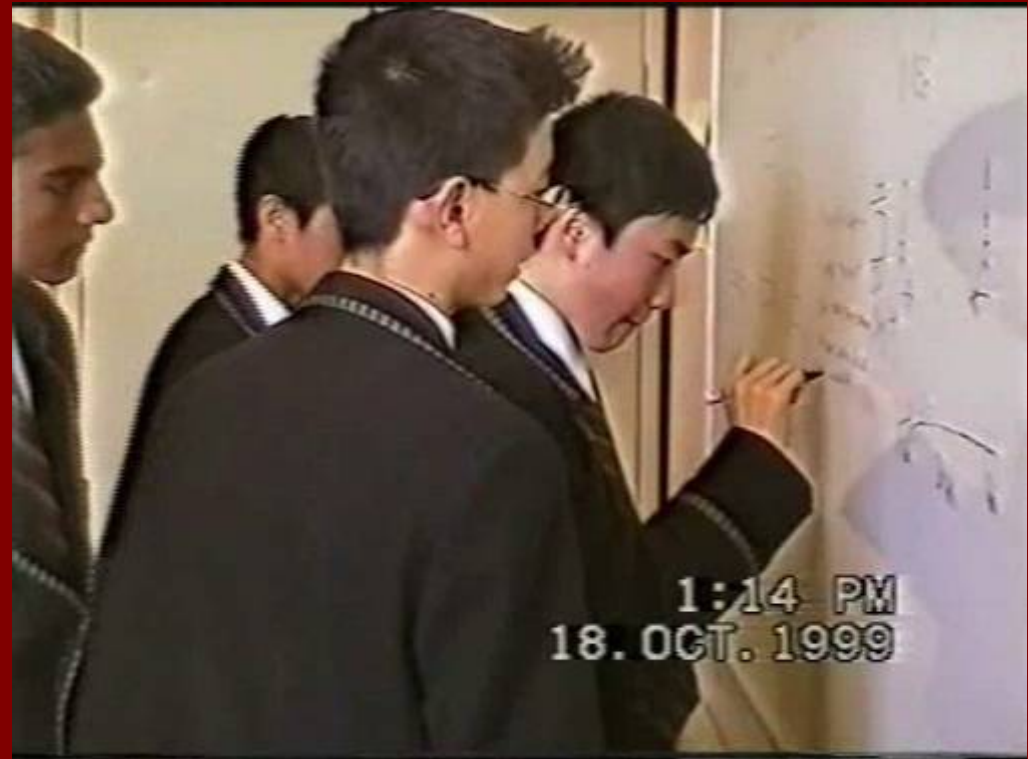
A truck which is 15 metres long drives on a straight stretch of road at a constant speed of 18 Km/hr. Two people are jogging on the footpath beside the road:

person A who is jogging in the same direction as the truck is driving and person B who is jogging towards the truck. Both are jogging at constant speeds.

The truck passes person A in 6 seconds.

32 seconds later it meets person B and passes him in 2 seconds.

How long after the truck passes person B will A and B meet?



.. or we can always recall old favourites

Year 10 Maths Extension Group

Challenging Question 2

1. Two containers A and B sit side by side on the kitchen bench. Container A holds one litre of water and Container B holds one litre of cordial.

100ml of water is carefully poured from Container A into Container B and the mixture completely mixed.

100ml of the mixture from Container B is returned to Container A.

Which is the larger ratio;
water:cordial in Container A,
or cordial:water in Container B?



The Year 10 Mathematics Extension Program structure

The Year 10 program consists of sets of challenging questions of an open-ended nature which do not parallel the mathematics program.

The content for this year is:

Semester 1

- **Measurement: will the spectators fit on the MCG during a disaster?**
- **Ratio: mixing water and cordial**
- **Circles and measurement**
- **Investigating sporting draws**

Semester 2

- **Speed, distance and time (the Hong Kong question !!)**
- **Probability**
- **Coordinate geometry**
- **Stacking balls - geometry**

The Year 9 Mathematics Extension Program structure

The Year 9 program consists of sets of challenging closed questions based on the topics studied as part of the Mathematics Program.

The topics are:

Semester 1

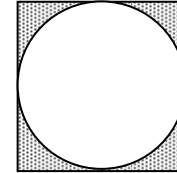
- **Number**
- **Algebra**
- **Measurement**
- **Exponentials and number**

Semester 2

- **Coordinate geometry**
- **Probability**
- **Geometry**
- **Algebraic inequalities**

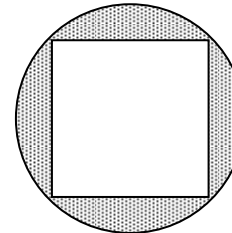
Year 9 Maths Extension Group: Challenging Question

1. (a) A circle with radius r cm is inscribed inside a square with radius.
- (i) Find the area of the square in terms of r and hence show that the shaded area is given by the expression: $r^2(4 - \pi)$.



- (ii) Write an expression for the fraction of the shape is shaded and evaluate it to two decimal places
- (b) A square is inscribed inside a circle with radius r cm.

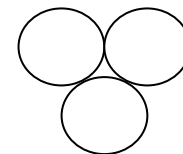
- (i) Express the edge length in terms of r and so find the area of the square in terms of r
- (ii) Show that the shaded area is given by the expression: $r^2(\pi - 2)$



- (iii) Write an expression for the fraction of the shape is shaded and evaluate it to two decimal places.
- (c) Use the answers from (a) (ii) and (b) (iii) to judge as to which shape fits better into the other shape.

2. Three identical circles are in contact with each other as is shown.

- (a) If each circle has a radius of r cm show that
- (i) the perimeter of the smaller shape formed in between the circles is given by the expression: πr cm
- (ii) the external perimeter (the perimeter of the outside of the shape) is given by the expression $5\pi r$
- (b) Show also that when the internal perimeter and the external perimeter are added the total is $6\pi r$.



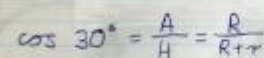
$\cos 60^\circ = \frac{r}{R}$
 $\cos 60^\circ = \frac{1}{2}$
 $\frac{1}{2} = \frac{r}{R}$
 $R = 2r$ ✓

$2R = 2(2r) = 4r$
 $2R = 2(2r) = 4r$
 $R(2 - \sqrt{3}) = \sqrt{3}r$
 $R = \frac{\sqrt{3}r}{2 - \sqrt{3}}$

$\frac{R}{r} = \frac{\sqrt{3}}{2 - \sqrt{3}}$
 $R = \frac{\sqrt{3}r}{2 - \sqrt{3}}$
 $R = \frac{\sqrt{3}r}{2 - \sqrt{3}}$
 $R = \frac{\sqrt{3}r}{2 - \sqrt{3}}$ ✓

Conclusion
 As you add more circles around the circle, more circles the same radius will be necessary per smaller. However, if the radius of the inner circle is known as well as the radius of one circle we can use the formula $R = r \frac{\sqrt{3}}{2 - \sqrt{3}}$ for figure out the radius of the outer circles. The formula we found before the for get the radius of the grid and only for get the same radius of circle in circle. The number a doubled that is more straightforward compared to right now. (2x) it was the only path the already known to find.

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 Date: *10/10/10*
 Page: *10*



$$\frac{\sqrt{3}}{2} = \frac{R}{R+r}$$

$$\begin{aligned} R\sqrt{3} + r\sqrt{3} &= 2R \\ r\sqrt{3} &= 2R - R\sqrt{3} \\ r\sqrt{3} &= R(2 - \sqrt{3}) \end{aligned}$$

$$\frac{7\sqrt{3}}{2-\sqrt{3}} = R$$

When $T = 1 \text{ cm}$

$$= \frac{1 \times \sqrt{3}}{2 \times \sqrt{3}} = \frac{\sqrt{3}}{2 \times \sqrt{3}} = 6.46 \text{ cm}$$

Reflection

By completing this task, I was able to learn the relationship between circles, ~~and~~ trigonometry and Pythagoras' Theorem. I also learnt a new formula that I can use to find the radius of a circle that surrounds another circle.

For a polygon of n sides,
 sum of interior angles $= (2n-4) \times 90^\circ$
 $= (n-2) \times 180^\circ$

So each angle in a shape = $\left(\frac{n-2}{2} \times 180^\circ\right)$

We use $\frac{1}{2}$ of this angle

$$\begin{aligned} \text{Angle} &= \frac{1}{2} \left(\frac{180n - 360}{n} \right) \\ &= \frac{90n - 180}{n} = \frac{90(n-2)}{n} \end{aligned}$$

$$\cos \theta = \frac{E}{K + T}$$

$$R \cos \theta + r \cos \theta = R$$

$$r \cos \theta = R(1 - \cos \theta)$$

$$R = \frac{r \cos \theta}{1 - \cos \theta}$$

$$R = \frac{r \cos \left\{ \frac{90(x-2)}{n} \right\}}{1 - \cos \left\{ \frac{90(x-2)}{n} \right\}}$$

By completing this task, I was able to learn the relationship between circles, ~~and~~ trigonometry and Pythagoras' Theorem. I also learnt a new formula that I can use to find the radius of a circle that surrounds another circle.

Spill